Schawlow's Ruler

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An HeNe laser ($632.8\,\mathrm{nm}$) was reflected off of the etched graduated surfaces of a metal ruler to create a diffraction patterns on an opposing vertical surface. These diffraction patterns were used to measure the wavelength of the laser. The $0.5\,\mathrm{mm}$ etching produced a diffraction pattern that yields a measured laser wavelength of $(634.3 \pm 0.2) \times 10^{-9}\,\mathrm{m}$, result differs from known by $\approx 7.5\,\sigma$. The $1.0\,\mathrm{mm}$ etching produced a diffraction pattern that yielded a measured laser wavelength of $(634.9 \pm 0.2) \times 10^{-9}\,\mathrm{m}$, result differs from known by $\approx 10.5\,\sigma$. The $5.0\,\mathrm{mm}$ etching produced a diffraction pattern that yielded a measured laser wavelength of $(636.0 \pm 0.2) \times 10^{-9}\,\mathrm{m}$, result differs from known by $\approx 16.0\,\sigma$.

Supervising Instructor: Thomas Colbert

I. INTRODUCTION

The equally-spaced grooves in the steel ruler create plateaus for reflection of the monochromatic light in constructive and destructive interference patterns (see Figure 1).

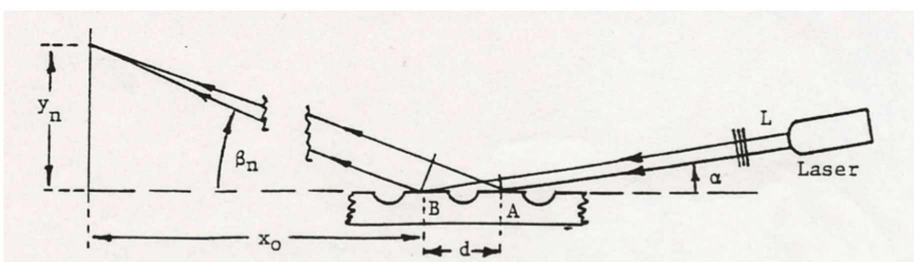

Figure 1: Interference Pattern. Source: Anonymous Handout[1].

It is important to notice that there is a difference between etched surface and stamped surfaces. A stamped surface will not have flat plateaus, but a raised "dimple" will exist on both sides of the stamped graduation. This dimple may disrupt the interference patterns to a certain extent.

The angle of the laser to the steel ruler must be fairly shallow, a couple degrees at most. Keeping the angle shallow allows for the beam to reflect off of a larger area of the ruler, which in turn produces sharper points in the diffraction pattern. The distance between the first maximum, y_o, and any subsequent maximum, y_n, plus the distance from the vertical surface and the center of the reflected spot on the ruler can be used to determine the wavelength of the source light[2]. A fairly accurate approximate value can be obtained for λ with angles small values from the following equation:

$$y_n^2 \approx \frac{2n\lambda x_o^2}{d} + y_o^2 \tag{1}$$

For all angles λ can be calculated exactly:

$$y_n^2 = \left\{ \frac{1}{\frac{1}{\sqrt{x_o^2+y_o^2}} - \frac{n\lambda}{dx_o}} \right\}^2 - x_o^2 \tag{2}$$

II. EQUIPMENT

- (1x) Uniphase Model 1135p Helium Neon Laser ($\lambda = 632.8\,\mathrm{nm}$) with power supply
- (1x) Laser Mount with Vertical and Horizontal Translator
- (1x) Steel Ruler with Etched Measurements
- Optical breadboard, ruler mounting hardware
- Measuring apparatus to 5-meter length

III. EXPERIMENTAL PROCEDURE

The laser was initially set at a very shallow angle to the ruler ($< 1°$). The incident laser beam was then smeared over a long distance on the ruler with very indistinct edges. The laser platform was raised on one end so that the angle was increased to about $3°$. This shortened the the reflective area to a distinct elongated oval (see Figure 2). The distance from the wall (target surface) to the center of the elongated oval reflection was obtained ($x_o = 4.587 \pm 0.005\,\mathrm{m}$).

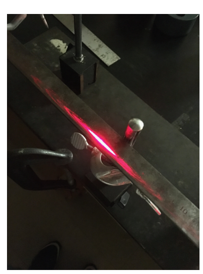

Figure 2: Reflection Pattern. A reflection with distinct boundaries allows for a more precise measurement of x_o.

Prior to taking any measurements, the steel ruler was adjusted so that it was parallel to the ground. The laser was horizontally translated so that the beam shined directly unto the wall, $-y_o$. The height of H_{-yo} above the floor was measured and subtracted from the measured height of the ruler H_r, then used to determine where the initial reflected point, H_{+yo}, should fall, by using the following equation:

$$H_{+yo} = 2(H_r - H_{-yo}) + H_{-yo} \tag{3}$$

The plane of the ruler was then adjusted so that $+y_o$ fell on the same spot as the calculated H_{+yo} (see Figure 3). The initial adjustment of the ruler changes H_r. Therefore the above procedure was repeated until measured H_{+yo} was less than 1 mm from calculated value. All diffraction maxima are measured from the center point between $-y_o$ and $+y_o$.

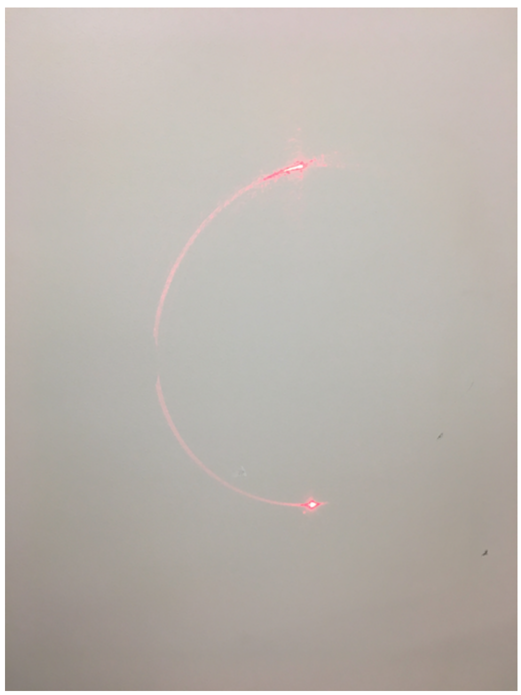

Figure 3: $-y_o$ **and** $+y_o$**.** A pen dot was placed on the paper where H_{+y_o} was located. During the procedure to set H_{+yo} it was discovered that if the laser beam was placed on the edge of the steel ruler, both $-y_o$ and $+y_o$ could be viewed simultaneously.

The full-setup is depicted in Figure 4:

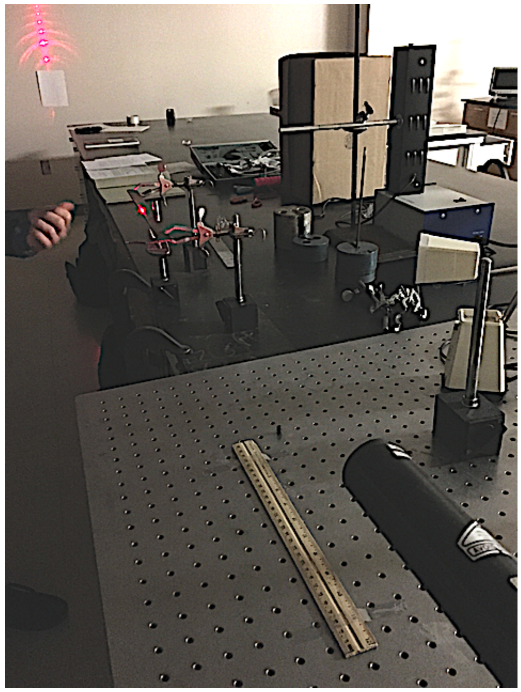

Figure 4: Full Experimental Setup. The end of the laser is shown in the bottom right corner. The beam reflection on the ruler and the diffraction pattern is located in the upper left corner.

In addition to the basic experiment, the steel ruler was flexed downward with a 5 kg weight to determine difference in the diffraction pattern from a flat ruler (see Figure 5).

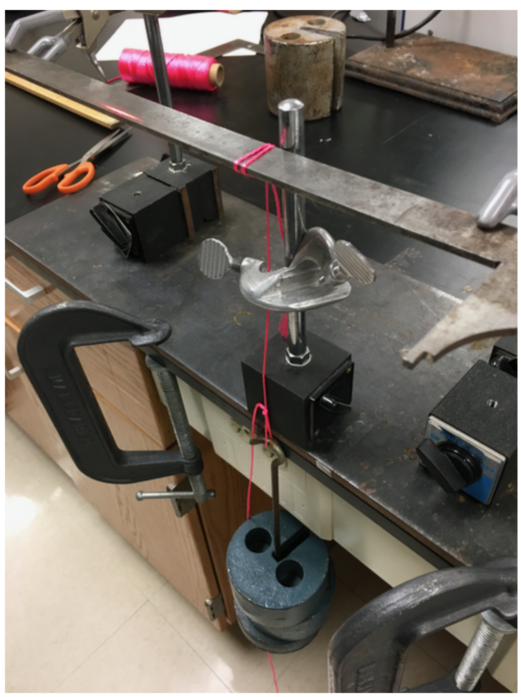

Figure 5: Bowed Ruler. The weight was shifted toward one end of the ruler and the laser beam to the other to prevent the string from interfering with the laser beam.

IV. ANALYSIS

Data was taken on three different scales on the steel ruler: $0.5\,\mathrm{mm}$, $1.0\,\mathrm{mm}$, and $5.0\,\mathrm{mm}$ (see Appendix 1, Table 3). Initial assessments of wavelengths from $n = 1$ and $n = 12$ of each set using the small angle approximation (see Equation 1) are found in Table 1. These values were used to determine validity of the data. The actual value of $632.8\,\mathrm{nm}$ falls within the error range of all calculated λ's. The error for x_0 was estimated at $\pm 0.005\,\mathrm{m}$ and the errors for y_n values were estimated $\pm 0.003\,\mathrm{m}$.

Table 1: Wavelength Approximations

Ruler Scale	n	Wavelength
0.5 mm	1	643 ± 13 nm
0.5 mm	12	645 ± 13 nm
1.0 mm	1	643 ± 13 nm
1.0 mm	12	641 ± 13 nm
5.0 mm	1	643 ± 13 nm
5.0 mm	12	643 ± 13 nm

Linear fit approximations using Equation 1 produced consistently high values (see Table 2). The closest approach to the accepted value was a low index in each data set. As indexes increase the approximate increases in value and departs from the accepted value.

Table 2: Wavelength Approximations

y_n, λ $d = 0.5$ **mm**	y_n, λ $d = 1.0$ **mm**	y_n, λ $d = 5.0$ **mm**
$y_n = 2$, $\lambda = 640.3 \pm 1.1$ nm	$y_n = 4$, $\lambda = 640.0 \pm 0.7$ nm	$y_n = 20$, $\lambda = 639.8 \pm 0.6$ nm
$y_n = 3$, $\lambda = 639.8 \pm 0.6$ nm	$y_n = 5$, $\lambda = 639.6 \pm 0.5$ nm	$y_n = 21$, $\lambda = 641.2 \pm 1.0$ nm
$y_n = 4$, $\lambda = 640.2 \pm 0.4$ nm	$y_n = 6$, $\lambda = 640.6 \pm 0.4$ nm	$y_n = 22$, $\lambda = 641.5 \pm 0.9$ nm
$y_n = 5$, $\lambda = 641.0 \pm 0.5$ nm	$y_n = 7$, $\lambda = 640.6 \pm 0.6$ nm	$y_n = 23$, $\lambda = 641.5 \pm 0.9$ nm
$y_n = 6$, $\lambda = 640.8 \pm 0.3$ nm	$y_n = 8$, $\lambda = 640.6 \pm 0.5$ nm	$y_n = 24$, $\lambda = 640.8 \pm 0.9$ nm
$y_n = 7$, $\lambda = 642.0 \pm 0.7$ nm	$y_n = 9$, $\lambda = 641.1 \pm 0.5$ nm	$y_n = 25$, $\lambda = 640.7 \pm 0.8$ nm
$y_n = 8$, $\lambda = 642.7 \pm 0.6$ nm	$y_n = 10$, $\lambda = 641.3 \pm 0.4$ nm	$y_n = 26$, $\lambda = 640.4 \pm 0.8$ nm
$y_n = 9$, $\lambda = 643.9 \pm 0.8$ nm		$y_n = 27$, $\lambda = 640.6 \pm 0.7$ nm
$y_n = 10$, $\lambda = 645.1 \pm 0.9$ nm		$y_n = 28$, $\lambda = 640.8 \pm 0.7$ nm
		$y_n = 29$, $\lambda = 641.3 \pm 0.7$ nm
		$y_n = 30$, $\lambda = 641.2 \pm 0.7$ nm

The above calculated linear fit approximations were based on Equation 1 using measured x_o, y_o, d, and y_n values. The linear fit approximations do not appear to have any overlap with the exact fit calculations based on Equation 2, using measured x_o, y_o, d, and y_n values and holding the fit route close to the expected $\lambda = 632.8\,\text{nm}$. The exact fits were much closer to the accepted value and much more precise (see Figures 6, 7, and 8). Measurements were not taken at 10 mm scale due to overlapping of maxima points.

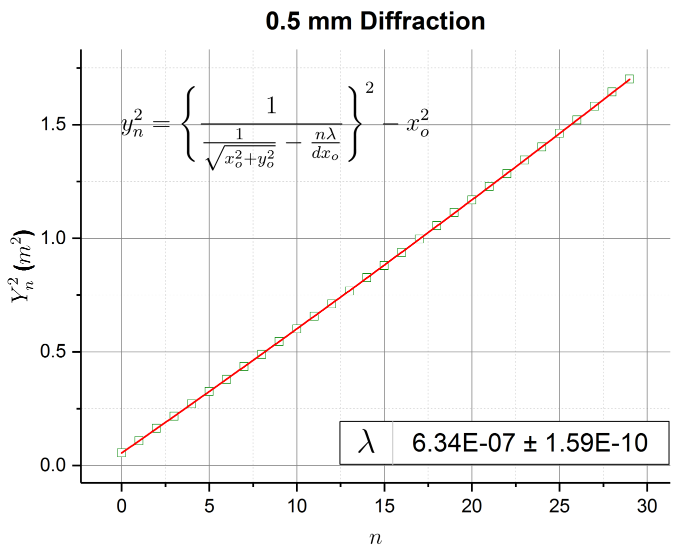

Figure 6: 0.5 mm Diffraction Calculations. Y_n^2 vs n, fit to λ while holding x_o, y_o, and d fixed.

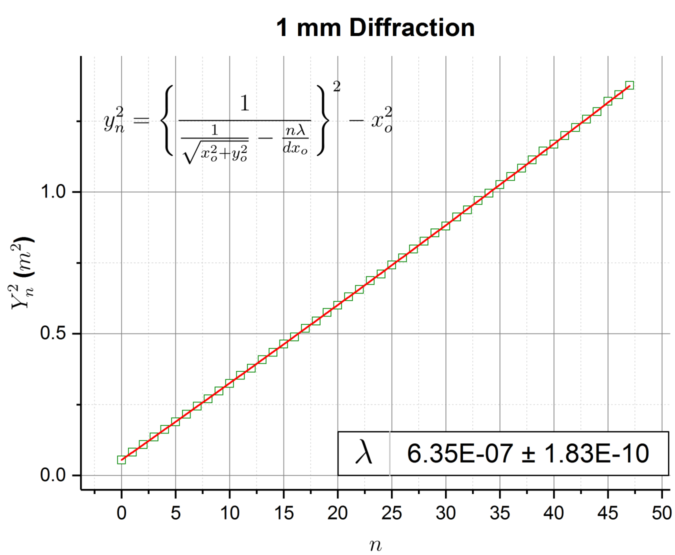

Figure 7: 1.0 mm Diffraction Calculations. Y_n^2 vs n, fit to λ while holding x_o, y_o, and d fixed.

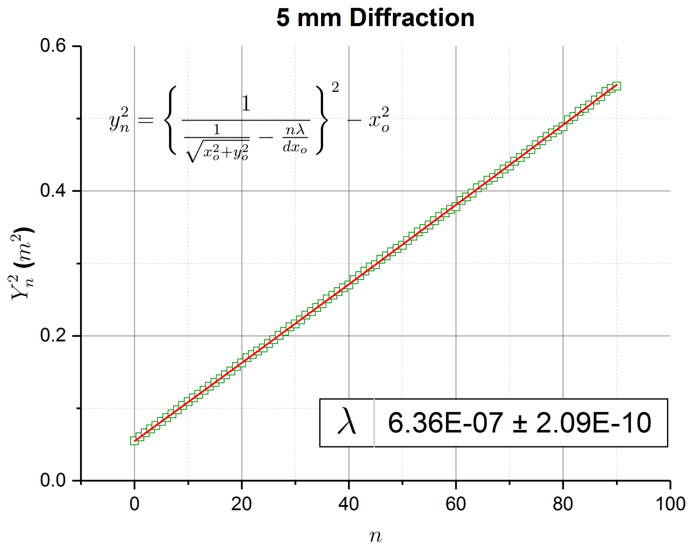

Figure 8: 5.0 mm Diffraction Calculations. Y_n^2 vs n, fit to λ while holding x_o, y_o, and d fixed.

The differences of estimated values from the expected value are so high largely because the estimated error on the fit curve is very low. However, if x_0 and y_0 are increased 0.004 and 0.002 (within the error estimates noted above), the calculated exact fit value for the 0.5 mm scale is $632.7 \pm 0.2\,\text{nm}$. Changing fit parameters did not yield reportable results. This clearly shows that exact fit value falls within the range of values calculated with raw uncertainties.

Addition of the $5\,\text{kg}$ weight to the scale did change the visual appearance of the maximum diffraction points (see Figure 9) and therefore make measurements and calculations more complex.

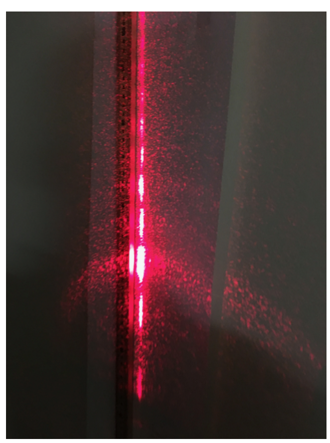

Figure 9: Diffraction Pattern of Bowed Ruler. The bowed ruler has elongated the points into bars. The center point of each bar was determined and used for calculations.

The calculated value of the wavelength using a bowed ruler is $639.8 \pm 0.6\,\mathrm{nm}$, much higher than values calculated with the flat ruler (see Figure 10). Moreover, this calculation assumes that the ruler is flat when instead it is bowed.

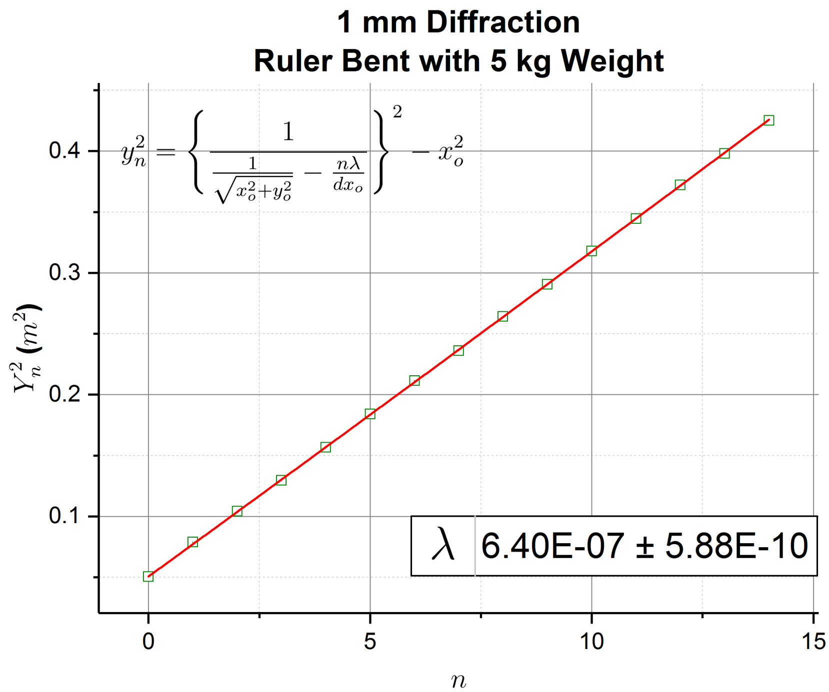

Figure 10: 1.0 mm Diffraction Calculations with 5 kg Weight Hanging from Ruler. Owing to the elongated maxima shown in Figure 9, the actual uncertainty may be orders of magnitude larger.

V. DISCUSSION

Calculated values using exact fits were precise, which gave large σ values. However, when estimated errors were taken into account, the actual value of $632.8\,\mathrm{nm}$ fell into the range of estimated errors. The values $634.3 \pm 0.2\,\mathrm{nm}$ for 0.5 mm diffraction pattern, $634.9 \pm 0.2\,\mathrm{nm}$ for the 1.0 mm diffraction pattern, and $636.0 \pm 0.2\,\mathrm{nm}$ for the 5.0 mm diffraction pattern differ only 0.23%, 0.33%, and 0.50% (respectively) from the accepted value.

VI. REFERENCES

(1) Anonymous; *Experiment 5: Interference of Light*

(2) Schawlow, A. L.; *Measuring the Wavelength of Light with a Ruler*, **June 1965**, American Journal Physics

A. EXPERIMENTAL DATA

Table 3: Full Data Set, Source: Student Lab Notebook

0.5 mm scale (± 0.003 m)	1.0 mm scale (± 0.003 m)	5.0 mm scale (± 0.003 m)	5.0 mm scale (cont'd) (± 0.003 m)	1.0 mm scale with 5 kg weight (± 0.003 m)
y_0 0.243	y_0 0.243	y_0 0.234	y_{48} 0.562	y_0 0.225
y_1 0.330	y_1 0.286	y_1 0.246	y_{49} 0.566	y_1 0.281
y_2 0.403	y_2 0.330	y_2 0.257	y_{50} 0.570	y_2 0.323
y_3 0.465	y_3 0.368	y_3 0.267	y_{51} 0.576	y_3 0.360
y_4 0.520	y_4 0.403	y_4 0.276	y_{52} 0.581	y_4 0.396
y_5 0.570	y_5 0.435	y_5 0.286	y_{53} 0.586	y_5 0.429
y_6 0.615	y_6 0.465	y_6 0.295	y_{54} 0.590	y_6 0.460
y_7 0.659	y_7 0.494	y_7 0.304	y_{55} 0.594	y_7 0.486
y_8 0.699	y_8 0.520	y_8 0.313	y_{56} 0.599	y_8 0.514
y_9 0.738	y_9 0.546	y_9 0.322	y_{57} 0.604	y_9 0.539
y_{10} 0.775	y_{10} 0.570	y_{10} 0.330	y_{58} 0.608	y_{10} 0.564
y_{11} 0.810	y_{11} 0.594	y_{11} 0.338	y_{59} 0.612	y_{11} 0.587
y_{12} 0.843	y_{12} 0.615	y_{12} 0.345	y_{60} 0.615	y_{12} 0.610
y_{13} 0.876	y_{13} 0.639	y_{13} 0.353	y_{61} 0.622	y_{13} 0.631
y_{14} 0.909	y_{14} 0.659	y_{14} 0.361	y_{62} 0.626	y_{14} 0.652
y_{15} 0.938	y_{15} 0.681	y_{15} 0.368	y_{63} 0.630	
y_{16} 0.968	y_{16} 0.699	y_{16} 0.375	y_{64} 0.636	
y_{17} 0.998	y_{17} 0.720	y_{17} 0.382	y_{65} 0.639	
y_{18} 1.027	y_{18} 0.738	y_{18} 0.389	y_{66} 0.644	
y_{19} 1.055	y_{19} 0.758	y_{19} 0.397	y_{67} 0.647	
y_{20} 1.081	y_{20} 0.775	y_{20} 0.403	y_{68} 0.651	
y_{21} 1.108	y_{21} 0.794	y_{21} 0.412	y_{69} 0.656	
y_{22} 1.133	y_{22} 0.810	y_{22} 0.417	y_{70} 0.659	
y_{23} 1.159	y_{23} 0.829	y_{23} 0.423	y_{71} 0.664	
y_{24} 1.184	y_{24} 0.843	y_{24} 0.428	y_{72} 0.668	
y_{25} 1.209	y_{25} 0.862	y_{25} 0.435	y_{73} 0.672	
y_{26} 1.233	y_{26} 0.876	y_{26} 0.441	y_{74} 0.676	
y_{27} 1.257	y_{27} 0.894	y_{27} 0.448	y_{75} 0.681	
y_{28} 1.282	y_{28} 0.909	y_{28} 0.454	y_{76} 0.685	
y_{29} 1.304	y_{29} 0.925	y_{29} 0.461	y_{77} 0.689	
	y_{30} 0.938	y_{30} 0.465	y_{78} 0.693	
	y_{31} 0.955	y_{31} 0.471	y_{79} 0.696	
	y_{32} 0.968	y_{32} 0.478	y_{80} 0.699	
	y_{33} 0.985	y_{33} 0.483	y_{81} 0.706	
	y_{34} 0.998	y_{34} 0.489	y_{82} 0.709	
	y_{35} 1.013	y_{35} 0.494	y_{83} 0.714	
	y_{36} 1.027	y_{36} 0.501	y_{84} 0.717	
	y_{37} 1.041	y_{37} 0.506	y_{85} 0.720	
	y_{38} 1.055	y_{38} 0.511	y_{86} 0.725	
	y_{39} 1.070	y_{39} 0.516	y_{87} 0.728	
	y_{40} 1.081	y_{40} 0.520	y_{88} 0.733	
	y_{41} 1.095	y_{41} 0.527	y_{89} 0.736	
	y_{42} 1.108	y_{42} 0.532	y_{90} 0.738	
	y_{43} 1.121	y_{43} 0.538		
	y_{44} 1.133	y_{44} 0.543		
	y_{45} 1.149	y_{45} 0.546		
	y_{46} 1.159	y_{46} 0.553		
	y_{47} 1.173	y_{47} 0.557		