Thermal Expansion of BK-7 Glass

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A HeNe laser beam is aimed through a small aperture upon the center of a BK-7 glass lens that is heated from room temperature to $\approx 65°C$. Measurements of changes in the resulting Airy disk and temperature are used to calculate the thermal expansion coefficient of the BK-7 optical glass. The coefficient of thermal expansion was calculated at $(8.38 \pm 0.22) \times 10^{-6}$, a difference of $0.4\sigma_\alpha$ from a reported value of 8.3×10^{-6} (in the temperature range of this experiment).

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I. INTRODUCTION

An Airy disk is formed when a HeNe laser beam is reflected from the front and read surface of a lens[1]. The disk is caused by differences in the path length of each beam (front reflected beam and rear reflected beam). As the lens is heated it expands further changing the path length difference, and to a much lesser degree the angles of incidence and reflection, and index of refraction[2]. The result of heating causes the path length to increase thereby making the rings of the disk to move toward the center. Cooling produces the opposite affect.

During the experiment drifting of the Airy disk is observed. The cause of drifting is surmised to be related to the position of the incident beam compared to the location of the optic center. The further away from the center, the more the drift. The tendency of drift for most of the trials performed is upward. Drifting impacts fringe counting by moving more fringes over a fixed point than have actually passed into the center of the pattern. Therefore, an offset was calculated for each trial and subtracted from the final count. Drifting may be reduced by using optics with optically flat, parallel surfaces[3].

Because the index of refraction is known not to change significantly in the target temperature range($\approx 20° - 60°$ C), for this experiment, the index of refraction is considered constant ($n = 1.51509$), and the analysis to determine the coefficient of expansion is treated as linear:

$$\alpha = \frac{\Delta L}{L_o \Delta T} \qquad \text{Source: [6]} \tag{1}$$

$$\Delta L = \frac{\Delta m \lambda}{2n} \qquad \text{Source: [6]} \tag{2}$$

$\Delta m =$ (passing ring count)

$\lambda = 632.8\,\text{nm}$ (HeNe laser wavelength)

$\Delta L =$ (change in lens width)

$L_o = (0.01003 \pm 0.00002)$ m (lens thickness)

$\Delta T =$ (change in lens temperature)

$n = 1.51509$ (index of refraction)

A positive α indicates an increase in material dimensions[2]. The standard approach is to treat thermal expansion non-linearly, by determining the change in the index of refraction with respect to temperature[2][3][4], referred to as the thermo-optic coefficient[3], then relating to thermal expansion coefficient:

$$n = n_o + \frac{dn}{dT}\Delta T \qquad \text{Source: [1]} \tag{3}$$

$$\frac{dn}{dT} = \frac{\lambda}{2L_o \Delta T} \qquad \text{Source: [2]} \tag{4}$$

$$\frac{dn}{dT} + (n-1)(1+v)\alpha = 0 \qquad \text{Source: [2]} \tag{5}$$

Equation 3 (above), owing to its similarity to equation 1 (above), provides a potential pathway for determining the thermo-optic coefficient from experimental data. However, the experimental setup used is not sufficiently precise to capture the change in lower significant digits. The change in refractive index per degree temperature would be less than noise generated by systematic error inherent in the experimental design.

Material properties which can satisfy Equation 4 (above), are purported to have minimal thermal lensing, an effect where thermal expansion and contraction changes the shape of an optic and thereby, the path of light through the optic[2]. This equation relates thermo-optic coefficient, Poisson's ratio (v), and thermal coefficient of expansion. Poisson's ratio is a property that measures mechanical strain. Exploration of these properties are outside the scope of this paper.

II. EXPERIMENTAL PROCEDURE

A BK-7 lens with a central thickness of $L_o = (0.01003 \pm 0.00002)$ m (measurement provided[7]) is mounted in a standard plastic mount inside a vertical heating duct. A hole was cut in the duct for the HeNe laser beam to reflect from the front and rear surface of the lens and exit the duct along the entrance path. A electric hot air blower is mounted to discharge hot air into the duct to create a uniform temperature zone around the lens (see Figure 1 below).

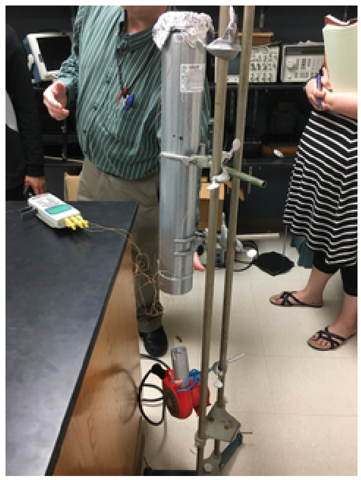

Figure 1: Experimental Setup. The HeNe laser beam enters duct from the left and exits along same path.

The incoming laser beam was focused through a hole in a box into the duct. The duct was positioned to allow the laser to reflect the HeNe laser beam from the front and rear of the center of the lens to create an Airy disk. Focus was aligned to move the center of this pattern exactly along the laser path (see Figure 2 below) to mitigate against the disk drifting.

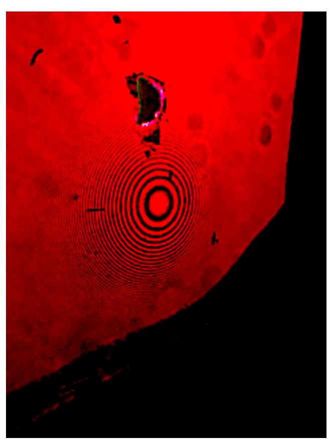

Figure 2: Airy disk. The hole is incoming path of the HeNe laser beam. The diffraction pattern depicted here is purposely lowered from the hole for the sake of clarity in the picture.

Inside the heat duct are several electronic temperature sensors to monitor the temperature of the ambient air and surfaces of the BK-7 glass. The initial temperature of the glass is recorded. The hot air blower is set to provide a slow, uniform rise in temperature with a maximum temperature at ≈ 65 (to prevent plastic lens mount from melting). The maximum temperature was recorded after the temperature plateaued for a period of time to ensure uniform internal lens temperature.

A video camera was set to record the changes in the Airy disk as the hot air was applied to the duct. The rings were observed to move toward the center during heating and away from the center during cooling. The videos were replayed and rings were counted as each passed a mark placed on the paper.

Enhancements to the experiment could be made by using a frequency stabilized HeNe with a collimater and thicker material[5][6]. Using an oven would require a much more complicated optical system[2]. An equipment list for this experimental setup is provided in Appendix 1.

III. ANALYSIS

Six trials were conducted where the Airy disk fringes were counted as they moved inward and the overall change in temperature recorded. The thermal expansion coefficient was calculated for each trial (see Table 1 below), then averaged for all trials. Large fringe count uncertainties are based on gross approximations of ring pattern drift distance. Drift distance was measured and an equivalent number of fringes was estimated to have passed the mark in this distance. This number of fringes was used as a negative offset correction for the drift. This offset estimation was a source of large error compared to other values obtained in the experiment. Drifting of the Airy disk is presumed to be caused by thermal expansion of lens mounts, and is considered largely unpreventable/uncorrectable in chosen experimental setup. Due to use of self-calibrating temperature sensors, temperature uncertainties were considered insignificant compared to fringe count uncertainties.

Table 1: Experimental Trials

Trial	Δm	m offset	Δm corrected	ΔT	α
1	19.5	-3.5 ± 0.5	16.0 ± 0.5	$41.45°C$	$(8.04 \pm 0.25) \times 10^{-6}$
2	20.0	-3.5 ± 0.5	16.5 ± 0.5	$41.05°C$	$(8.37 \pm 0.25) \times 10^{-6}$
3	18.5	-3.0 ± 0.5	15.5 ± 0.5	$38.05°C$	$(8.48 \pm 0.27) \times 10^{-6}$
4	17.25 ± 0.1	0.0	17.25 ± 0.1	$41.25°C$	$(8.71 \pm 0.05) \times 10^{-6}$
5	19.0	-2.5 ± 0.5	16.5 ± 0.5	$40.90°C$	$(8.40 \pm 0.25) \times 10^{-6}$
6	20.0	-3.5 ± 0.5	16.5 ± 0.5	$41.60°C$	$(8.26 \pm 0.25) \times 10^{-6}$

Table 1: Experimental Trials. Six trials were conducted. Trial four used an expanded data set that increased precision compared to the other trials which only used beginning and end points. Trial four was chosen for expansion because no drift was observed.

The average coefficient of thermal expansion is $(8.38 \pm 0.22) \times 10^{-6}$, a difference of $0.4\sigma_\alpha$ from a reported value of 8.3×10^{-6}[3] (in the temperature range of this experiment). While it is known that the thermal expansion coefficient is non-linear over extended ranges, a linear fit was performed on temperature values at 0.5 intervals of fringe counts from the fourth trial (see Table 2 below).

Table 2: Fourth Trial Data

Fringe Count	T	Fringes Cont'd	T
0.00	$23.90°C$	9.33	$45.50°C$
0.33	$24.25°C$	9.83	$46.80°C$
0.83	$25.35°C$	10.33	$48.20°C$
1.33	$26.45°C$	10.83	$49.35°C$
1.83	$27.95°C$	11.33	$50.45°C$
2.33	$29.35°C$	11.83	$51.45°C$
2.83	$30.50°C$	12.33	$52.65°C$
3.33	$31.60°C$	12.83	$53.70°C$
3.83	$32.70°C$	13.33	$54.90°C$
4.33	$34.10°C$	13.83	$56.15°C$
4.83	$35.30°C$	14.33	$57.15°C$
5.33	$36.50°C$	14.83	$58.25°C$
5.83	$37.60°C$	15.33	$59.25°C$
6.33	$38.90°C$	15.83	$60.65°C$
6.83	$39.90°C$	16.33	$61.75°C$
7.33	$40.90°C$	16.83	$62.95°C$
7.83	$42.00°C$	17.33	$64.10°C$
8.33	$43.10°C$	17.58	$64.70°C$
8.83	$44.20°C$		

Table 2: Fourth Trial Data. The trial began where the reference point was 0.33 fringes inward from nearest dark fringe and ended where the reference point was 0.25 fringes outward from nearest dark fringe.

The fit slope was used in place of $\Delta m/\Delta T$ in equation 1. The calculated coefficient of expansion based on the linear fit is $(8.93 \pm 0.03) \times 10^{-6}$, a difference of $21.4\sigma_\alpha$ from the reported value. Future experiments using narrower temperature ranges (subsets of the temperatures used in this experiment) should determine if a linear fit model will better approximate the reported value.

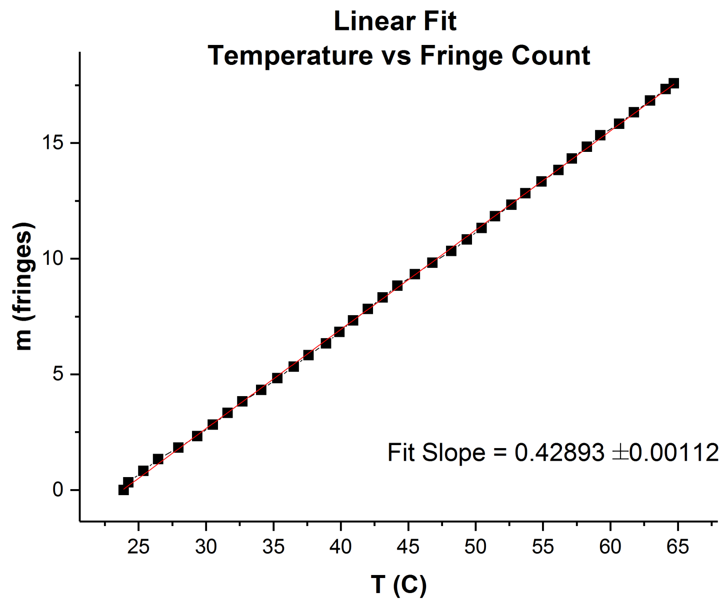

Figure 2: Linear Fit Line. Even though thermal expansion is non-linear, the individual points fall above and below at several places on the linear fit depicted in the graph.

IV. DISCUSSION

While the intent of this experiment is to show that coefficient of expansion could be measured with light, it has shortcomings that must be examined and addressed before the data obtained can be asserted with any validity. First, the experiment is based upon the assumption that coefficient of expansion is linear, whereas the standard treatment in other experiments show non-linearity. Second, the experimental setup evidences a crude assembly of components rather than a cohesive design–this promotes large corrective factors and systematic errors in analysis that are impossible to overcome. Despite these shortcomings, the average of a small number of trials was calculated only $0.4\sigma_\alpha$ from reported value of the target material (BK-7 optical glass), $\alpha = 8.3 \times 10^{-6}$[4].

V. REFERENCES

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VI. APPENDIX 1: EQUIPMENT

- (1x) Uniphase Model 1135p Helium Neon Laser ($\lambda = 632.8\,\text{nm}$) with power supply
- (1x) BK-7 Optical Glass Lens
- (1x) Laser Mount
- (1x) Lens Mount
- (1x) Electrical Heater
- (1x) Heating duct and mounting apparatus